6.300: Signal Processing

Tutorial Topic Filtering Images
Author Titus K. Roesler (tkr@mit.edu)

Filtering Images

We should be able to solve image processing problems in both the space and frequency domains.

Space Domain

Circularly-convolve the input $x[r,c]$ with the unit-sample response $h[r,c]$. Use superposition!

$$x[r,c] \rightarrow \boxed{h[r,c]} \rightarrow y[r,c] = \frac{1}{RC}(x \circledast h)[r,c]$$

Frequency Domain

Multiply the 2D DFT $X[k_r,k_c]$ by the 2D DFT $H[k_r,k_c]$ pointwise and then inverse transform.

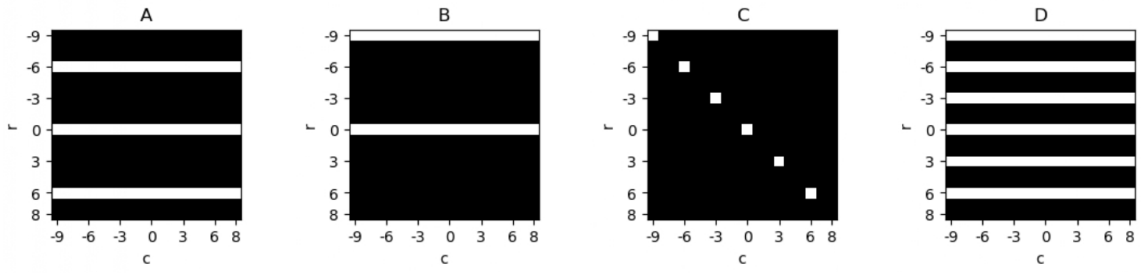
$$X[k_r,k_c] \rightarrow \boxed{H[k_r,k_c]} \rightarrow Y[k_r,k_c] = X[k_r,k_c]H[k_r,k_c]$$

While spatial-domain circular convolution and frequency-domain filtering are equivalent, some problems are more easily solved in one domain than the other.

Filtering Images

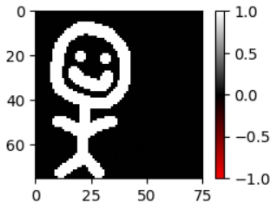
Which of the following images can be constructed by

- circularly convolving two of the other images
- inverse transforming the product of the DFTs of two images



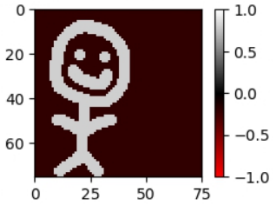
Stickmen

Let $X[k_r, k_c]$ represent the 2D DFT of the following image.

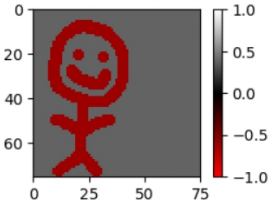


Which of A-F (if any) shows the iDFT of the real part of X?

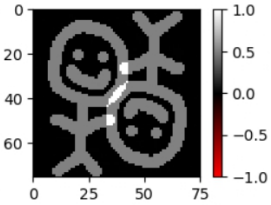
A



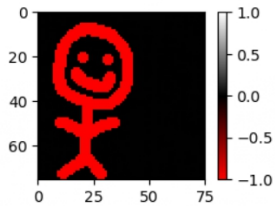
B



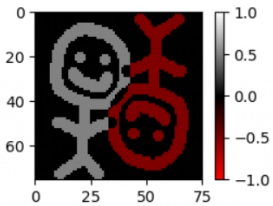
C



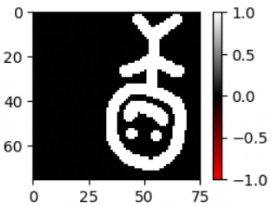
D



E



F



Stickmen

Refer to plots A, B, C, D, E, and F above.

- Which plot (if any) shows the inverse DFT (IDFT) of $\text{Re}\{X[k_r, k_c]\}$?
- Which plot (if any) shows the IDFT of $\text{Im}\{X[k_r, k_c]\}$?
- Which plot (if any) shows the IDFT of $j \cdot \text{Im}\{X[k_r, k_c]\}$?
- Which plot (if any) shows the IDFT of X after setting $X[0, 0] = 0$?
- Which plot (if any) shows the IDFT of X after multiplying every point by $e^{j\pi}$?
- Which plot (if any) shows the IDFT of X after multiplying every point *except* $X[0, 0]$ by $e^{j\pi}$?
- Which plot (if any) shows the IDFT of X after negating the phase of each point in X?

Note: You might find this problem a useful reference for a homework problem with cat pictures.