6.300: Signal Processing

Tutorial Topic Short-Time Fourier Transform
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Window Functions

A defining feature of the DFT is that signals we analyze have a finite length: N samples. The number of samples plays a key role in determining the trade-off between time resolution and frequency resolution.

We can restrict a signal $x[n]$ to have length N by multiplying pointwise by a window function $w[n]$.

$$\textbf{Windowing:}\ x_w[n] = x[n]w[n] \iff X_w(\Omega) = \tfrac{1}{2\pi}(X * W)(\Omega)$$

What is the effect of the window function $w[n]$ in the frequency domain?

Rectangular Window (Boxcar Window)

$$w_1[n] = \begin{cases} \frac{1}{2L-1} & 0 \le n < 2L-1 \\ 0 & \text{otherwise} \end{cases}$$

- Plot $w_1[n]$ against n.
- Determine the discrete-time Fourier transform $W_1(\Omega)$.
- Plot $|W_1(\Omega)|$ against Ω.

Triangular Window (Bartlett Window)

$$w_2[n] = \begin{cases} \frac{1}{L^2}(n+1) & 0 \le n < L \\ \frac{1}{L^2}(2L-n-1) & L \le n < 2L-1 \\ 0 & \text{otherwise} \end{cases}$$

- Plot $w_2[n]$ against n.
- Determine the discrete-time Fourier transform $W_2(\Omega)$.
- Plot $|W_2(\Omega)|$ against Ω.

Hann Window

$$w_3[n] = \begin{cases} \frac{1}{5}\sin^2\left(\frac{\pi(n+1)}{2L-1}\right) & 0 \le n < 2L-1 \\ 0 & \text{otherwise} \end{cases}$$

- Plot $w_3[n]$ against n.
- Determine the discrete-time Fourier transform $W_3(\Omega)$.
- Plot $|W_3(\Omega)|$ against Ω.

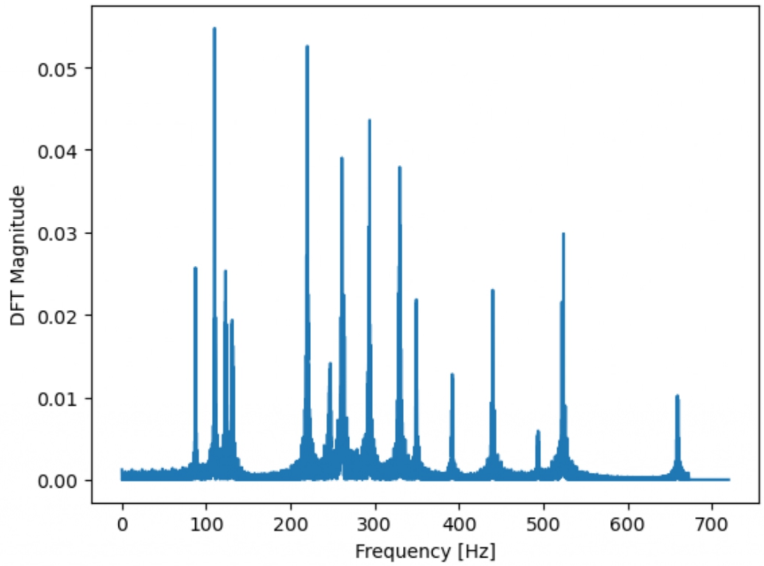
Window Function Comparison

Compare and contrast the plots of $|W_1(\Omega)|$, $|W_2(\Omega)|$, and $|W_3(\Omega)|$.
What are important differences?

Music Clip

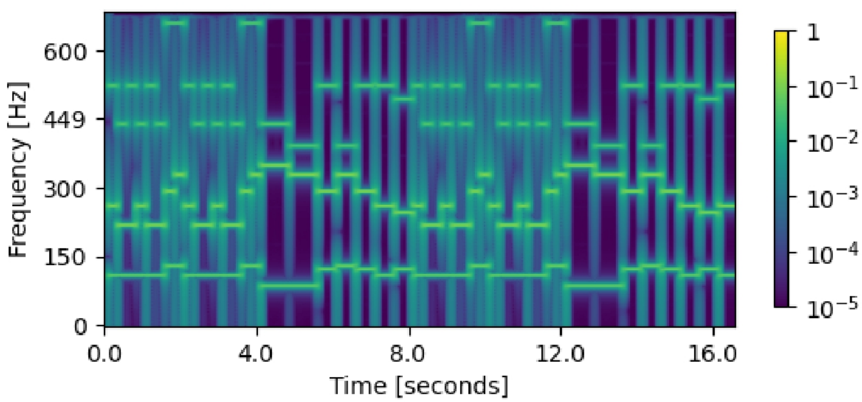
In lecture, we saw three representations for the same music clip.

The first was the magnitude of the DFT (shown below).



Music Clip

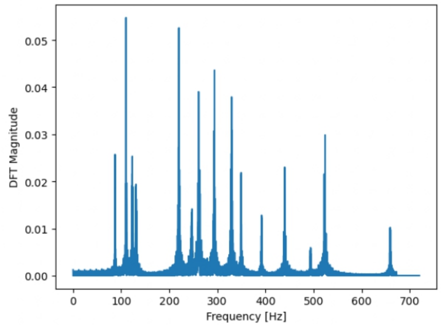
The second was the spectrogram.

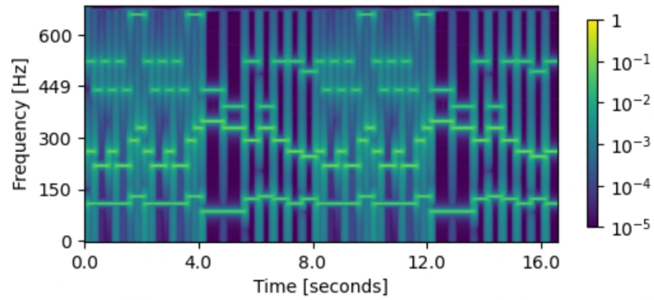


The third was the musical score.

Music Clip

Compare features of these representations.

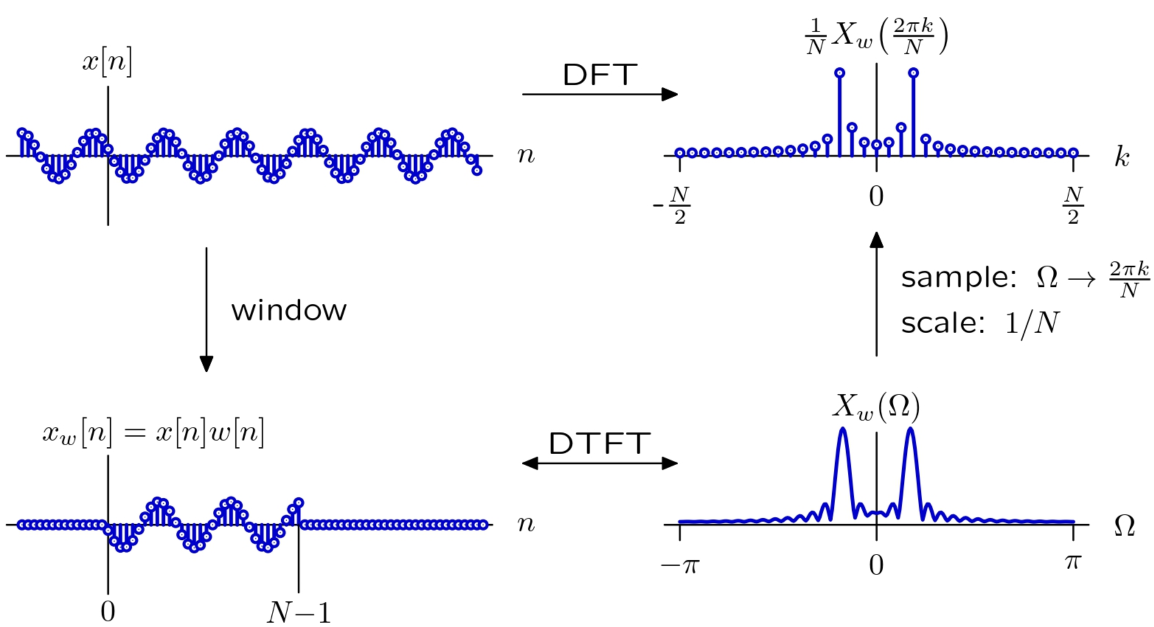




- What do these representations have in common?
  How are they different?
- Which representation has the best frequency resolution? Why?
- Which representation has the best time resolution? Why?
- The left panel has peaks with different heights. How do the different heights in the left panel correspond to features of the spectrogram?

Window Functions

A defining feature of the DFT is its finite length N, which plays a critical role in determining both time and frequency resolution.



The finite length constraint is equivalent to multiplication by a rectangular window. What would happen if we used a different type of window?

Windowing

Multiplying $x[n]$ by the window function $w[n]$ in time
corresponds to convolving the DTFT of $x[n]$ with the DTFT of $w[n]$ in frequency.

$$x_w[n] = x[n]w[n] \iff X_w(\Omega) = \frac{1}{2\pi}\big(X * W\big)(\Omega)$$