6.300: Signal Processing

Tutorial Topic Modulation and Communication Systems
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Communication Systems

A key problem in the design of any communication system is matching characteristics of the signal to those of the channel medium. Much of the current research in communications focuses on modifying signals to better accommodate constraints imposed by the channel medium. In lecture, we looked at simple channel-medium-matching strategies based on modulation. Modulation underlies virtually all matching schemes.

Modulation Property (Frequency Shift Property)

Multiplying a signal $x(t)$ by a time-varying complex exponential shifts the transform $X(\omega)$ in frequency.

$$x(t)e^{j\omega_c t} \iff \tfrac{1}{2\pi}(X(\omega) * 2\pi\delta(\omega-\omega_c)) = X(\omega-\omega_c)$$

Just as convolution in time corresponds to multiplication in frequency, multiplication in time corresponds to convolution in frequency.

Amplitude Modulation

Transmission

To transmit a real-valued message signal $x(t)$ at some carrier frequency, multiply $x(t)$ by the sinusoidal carrier signal $c(t)$ and transmit the modulated signal $y(t) = x(t)c(t)$.

Reception

We can recover the real-valued message signal $x(t)$ from the modulated signal $y(t)$ by multiplying $y(t)$ by the sinusoidal carrier signal $c(t)$ and then low-pass filtering.

$$c(t) = \cos(\omega_c t) = \tfrac{1}{2}e^{j\omega_c t} + \tfrac{1}{2}e^{-j\omega_c t}$$
$$y(t) = x(t)\,c(t) \iff Y(\omega) = \tfrac{1}{2\pi}(X * C)(\omega)$$
$$Y(\omega) = \underbrace{\tfrac{1}{2}X(\omega-\omega_c) + \tfrac{1}{2}X(\omega+\omega_c)}_{\text{copies of } X(\omega) \text{ shifted outward by } \omega_c}$$

Sinusoidal Modulation

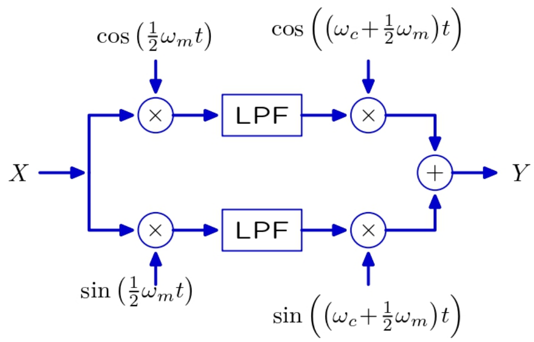
We examined amplitude modulation (AM) in class. Perhaps you've heard of frequency modulation (FM) or phase modulation (PM) — but you don't need to know these for this class, per se.

$$y(t) = A\cos(\omega t + \phi)$$

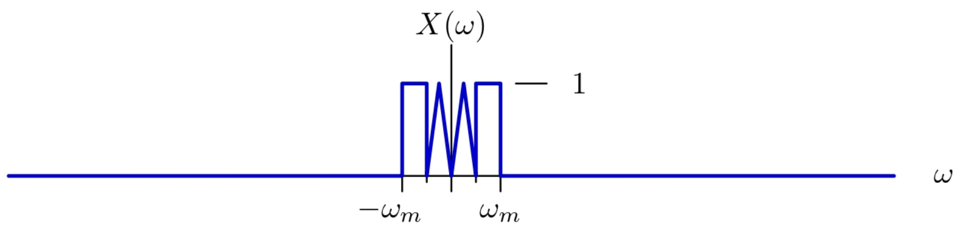
- amplitude modulation (AM) — time-varying amplitude $A = A(t)$
- frequency modulation (FM) — time-varying frequency $\omega = \omega(t)$
- phase modulation (PM) — time-varying phase $\phi = \phi(t)$

Bandwidth Conservation

Consider the following modulation scheme, where $\omega_c >> \omega_m$.



Assume that each lowpass filter (LPF) is ideal, with cutoff frequency $\omega_m/2$. Also assume that the input signal has the following Fourier transform.



Determine $Y(\omega)$.

Implementing a Bandpass Filter

The critical components of many modulation systems include bandpass filters, which are filters that pass only frequencies in a given range. Which of following systems implement a bandpass filter?