6.300: Signal Processing

Tutorial Topic LTI Systems
Author Titus K. Roesler (tkr@mit.edu)

Linear Time-Invariant Systems

A linear time-invariant (LTI) system is a system which is both linear and time-invariant.

$$x(t) \rightarrow \boxed{\textbf{LTI}} \rightarrow y(t) \qquad\qquad x[n] \rightarrow \boxed{\textbf{LTI}} \rightarrow y[n]$$

We will look at three equivalent representations of LTI systems in this class.

- differential equation (CT) or difference equation (DT)
- impulse response (CT) or unit-sample response (DT)
- frequency response

Linearity

A system is linear if and only if it is both additive and homogeneous.

Additivity

The response to a sum of inputs is a sum of the respective outputs.

$$\text{If } x_1[n] \rightarrow \boxed{\text{additive system}} \rightarrow y_1[n] \text{ and } x_2[n] \rightarrow \boxed{\text{additive system}} \rightarrow y_2[n],$$

$$\text{then } x[n] = x_1[n] + x_2[n] \rightarrow \boxed{\text{additive system}} \rightarrow y[n] = y_1[n] + y_2[n].$$

Homogeneity

Scaling the input correspondingly scales the output.

$$\text{If } x[n] \rightarrow \boxed{\text{homogeneous system}} \rightarrow y[n], \text{ then } \alpha \cdot x[n] \rightarrow \boxed{\text{homogeneous system}} \rightarrow \alpha \cdot y[n].$$

Linearity

The response to a sum of scaled inputs is a sum of the respective scaled outputs.

$$\text{If } x_1[n] \rightarrow \boxed{\text{linear system}} \rightarrow y_1[n] \text{ and } x_2[n] \rightarrow \boxed{\text{linear system}} \rightarrow y_2[n],$$

$$\text{then } x[n] = \alpha \cdot x_1[n] + \beta \cdot x_2[n] \rightarrow \boxed{\text{linear system}} \rightarrow y[n] = \alpha \cdot y_1[n] + \beta \cdot y_2[n].$$

Time-Invariance

Delaying (or advancing) the input delays (or advances) the output by the exact same duration of time.

$$\text{If } x[n] \rightarrow \boxed{\text{time-invariant system}} \rightarrow y[n], \text{ then } x[n - n_0] \rightarrow \boxed{\text{time-invariant system}} \rightarrow y[n - n_0].$$

Linearity and Time-Invariance

(a) Is this discrete-time system linear? Is this system time-invariant?

$$y[n] = x[-n]$$

(b) Is this discrete-time system linear? Is this system time-invariant?

$$y[n] = x[2n]$$

(c) Is this discrete-time system linear? Is this system time-invariant?

$$y[n] = |x[n]|$$

(d) Is this continuous-time system linear? Is this system time-invariant?

$$y(t) = \frac{dx(t)}{dt}$$

(e) Is this continuous-time system linear? Is this system time-invariant? **Hint:** Change of variables.

$$y(t) = \int_0^t x(\tau)d\tau$$

(f) Is this continuous-time system linear? Is this system time-invariant? **Hint:** Change of variables.

$$y(t) = \int_{-\infty}^t x(\tau)d\tau$$

(g) Is this continuous-time system linear? Is this system time-invariant? **Hint:** Change of variables.

$$y(t) = \int_t^{\infty} x(\tau)d\tau$$

(h) Is this continuous-time system linear? Is this system time-invariant?

$$y(t) = \int_{-\infty}^{\infty} x(\tau)d\tau$$