6.300: Signal Processing

Tutorial Topic Discrete-Time Fourier Transform
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Discrete-Time Fourier Transform (DTFT)

The discrete-time Fourier transform is the discrete-time analogue of the continuous-time Fourier transform — a Fourier transform for discrete-time signals. Infinitely-many discrete harmonics $k\Omega_0$ cluster infinitely-close together to form a continuous frequency spectrum: $k\Omega_0 \mapsto \Omega$.

Synthesis: $x[n] = \dfrac{1}{2\pi}\displaystyle\int_{2\pi} X(\Omega)e^{j\Omega n}d\Omega$ **Analysis:** $X(\Omega) = \displaystyle\sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

Discrete-Time Fourier Transform Pairs

$$\delta[n] \iff 1 \text{ (for all } \Omega) \qquad 1 \text{ (for all } n) \iff 2\pi\delta(\Omega \bmod 2\pi)$$

$$\delta[n-n_0] \iff e^{-j\Omega n_0} \qquad e^{j\Omega_0 n} \iff 2\pi\delta(\Omega - \Omega_0 \bmod 2\pi)$$

Discrete-Time Fourier Transform Properties

You fill this in! Many properties that we've seen in the context of Fourier series still hold true.
After you solve the problems on the other side of this sheet, write down the properties you just derived.

Fourier Series vs. Fourier Transform for Periodic Signals

Series: $f[n] \iff F[k]$ **Transform:** $f[n] \iff \displaystyle\sum_k \underbrace{2\pi F[k]\delta(\Omega - k\Omega_0)}_{\text{impulses at harmonics}}$

Discrete-Time Fourier Transform

(a) Determine $X_1(\Omega)$, the discrete-time Fourier transform (DTFT) of $x_1[n]$. Following this, sketch the magnitude $|X_1(\Omega)|$ and phase $\angle X_1(\Omega)$ over the interval $-\pi \leq \Omega \leq \pi$ on separate plots.

$$x_1[n] = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

(b) Determine $X_2(\Omega)$, the DTFT of $x_2[n]$. Following this, sketch the magnitude $|X_1(\Omega)|$ and phase $\angle X_1(\Omega)$ over the interval $-\pi \leq \Omega \leq \pi$ on separate plots. **Hint:** You ought to be able to re-use much of the work you did in **(a)** if you apply your knowledge of Fourier transform properties.

$$x_2[n] = x_1[n - n_0]$$

(c) Determine $X_3(\Omega)$, the DTFT of $x_1[n]$. Following this, fill in the blanks: A time-domain signal that is real and symmetric has a Fourier transform that is ________________ and ________________.

$$x_3[n] = \text{Symmetric}\{x_1[n]\}$$

(d) Determine $X_4(\Omega)$, the DTFT of $x_4[n]$. Following this, fill in the blanks: A time-domain signal that is real and antisymmetric has a Fourier transform that is ________________ and ________________.

$$x_4[n] = \text{Antisymmetric}\{x_1[n]\}$$

(e) Determine $X_5(\Omega)$, the DTFT of $x_5[n]$. **Hint:** Start by writing out the DTFT analysis equation. Following this, multiply both sides by j and differentiate both sides with respect to Ω.

$$x_5[n] = n x_1[n]$$

(f) Sketch $x_6[n]$ for $0 \leq n \leq 10$. Determine $X_6(\Omega)$, the DTFT of $x_6[n]$. **Hint:** Start by writing out the DTFT analysis equation. Following this, make a change of variables to simplify the summation.

$$x_6[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n = 0, 2, 3, 4, 6, 8, \ldots, \infty \\ 0 & \text{otherwise} \end{cases}$$

Inverse Discrete-Time Fourier Transform

(g) Determine the time-domain signal $x_7[n]$ whose discrete-time Fourier transform is $X_7(\Omega)$.

$$X_7(\Omega) = e^{-j3\Omega}$$