6.300: Signal Processing

Two-Dimensional Fourier Transforms

2D DFT: Analysis and Synthesis Equations

$$F[k_c, k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} f[r,c]e^{-j(k_r\frac{2\pi}{R}r+k_c\frac{2\pi}{C}c)}$$

$$f[r,c] = \sum_{k_r=0}^{R-1}\sum_{k_c=0}^{C-1} F[k_r,k_c]e^{j(k_r\frac{2\pi}{R}r+k_c\frac{2\pi}{C}c)}$$

Separability: $f[r,c] = f_{\mathcal{R}}[r]f_{\mathcal{C}}[c] \iff F_{\mathcal{R}}[k_r]F_{\mathcal{C}}[k_c]$

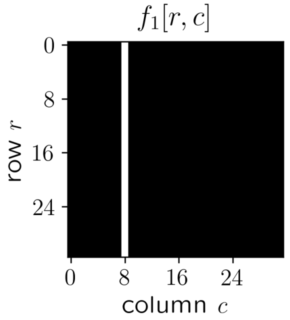
The 2D DFT of a vertical bar is a horizontal bar — and vice versa. The 2D DFT of an impulse train is an impulse train with inversely-proportional spacing.

Agenda for Recitation

- Two-dimensional discrete Fourier transform (2D DFT)

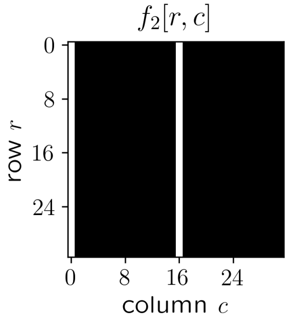
What questions do you have from lecture?

2D Discrete Fourier Transform



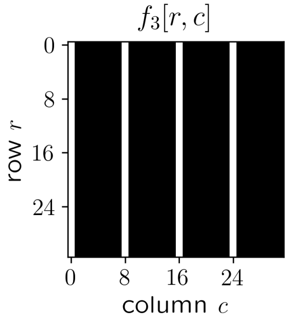
Consider the 2D signal $f_1[r,c]$ shown above. Assume that black corresponds to a value of zero and white corresponds to a value of one. Determine an expression for $F_1[k_r,k_c]$, the 2D DFT of $f_1[r,c]$.

2D Discrete Fourier Transform



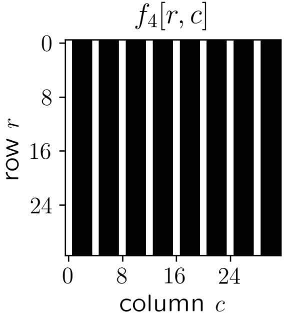
Consider the 2D signal $f_2[r, c]$ shown above. Assume that black corresponds to a value of zero and white corresponds to a value of one. Determine an expression for $F_2[k_r, k_c]$, the 2D DFT of $f_2[r, c]$.

2D Discrete Fourier Transform



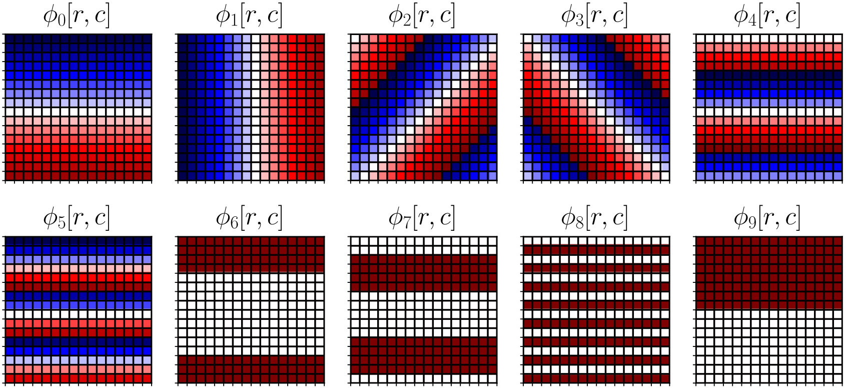
Consider the 2D signal $f_3[r,c]$ shown above. Assume that black corresponds to a value of zero and white corresponds to a value of one. Determine an expression for $F_3[k_r,k_c]$, the 2D DFT of $f_3[r,c]$.

2D Discrete Fourier Transform



Consider the 2D signal $f_4[r, c]$ shown above. Assume that black corresponds to a value of zero and white corresponds to a value of one. Determine an expression for $F_4[k_r, k_c]$, the 2D DFT of $f_4[r, c]$.

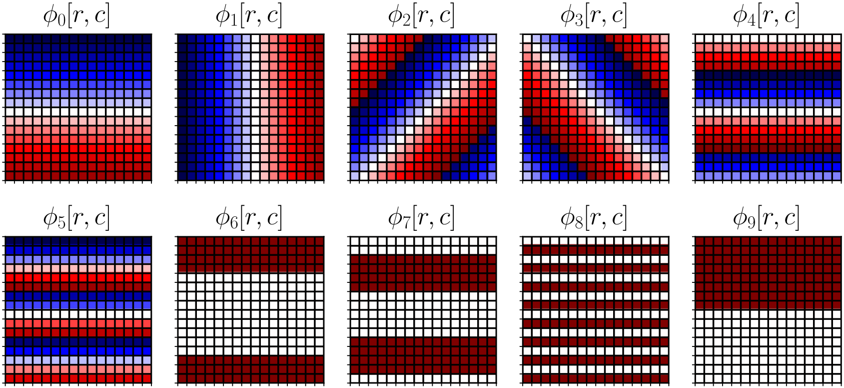
Plane Waves



The origin $(0,0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_0.

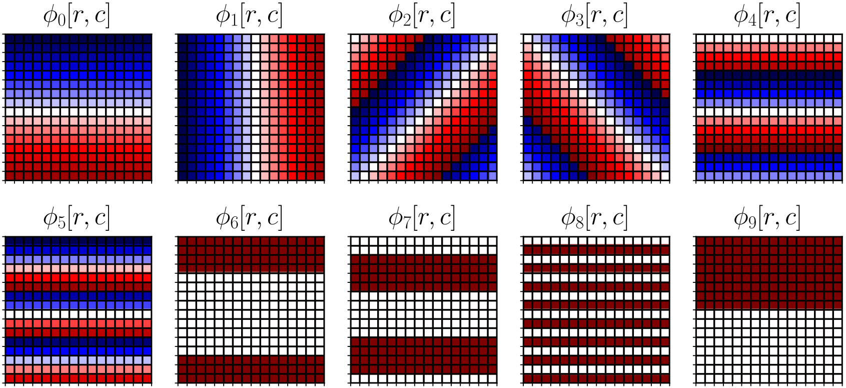
Plane Waves



The origin $(0, 0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_1.

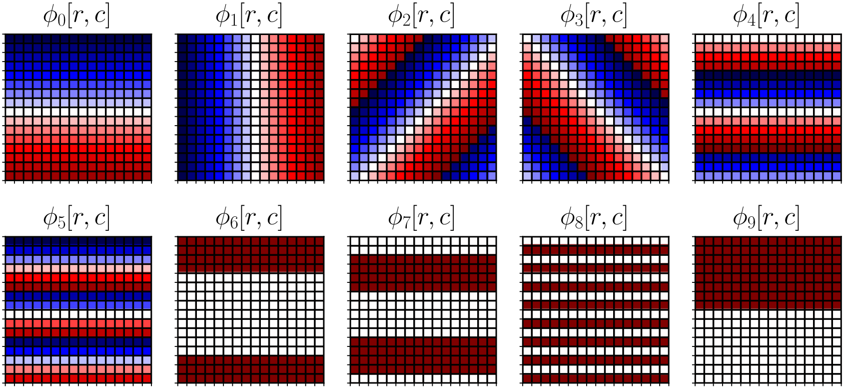
Plane Waves



The origin $(0,0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_2.

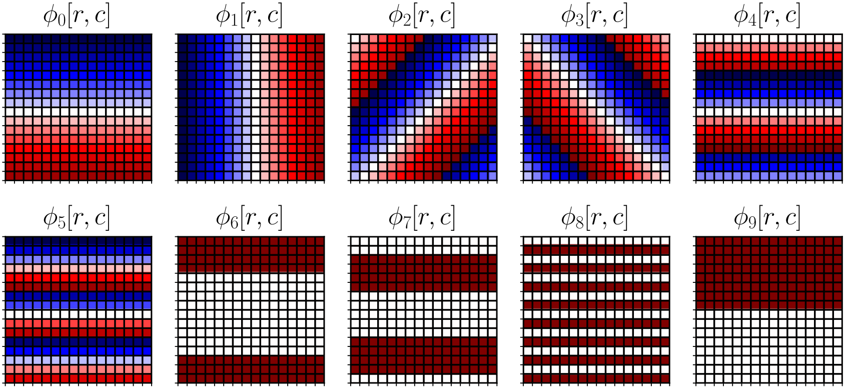
Plane Waves



The origin $(0, 0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

> Determine a waveform with angle ϕ_3.

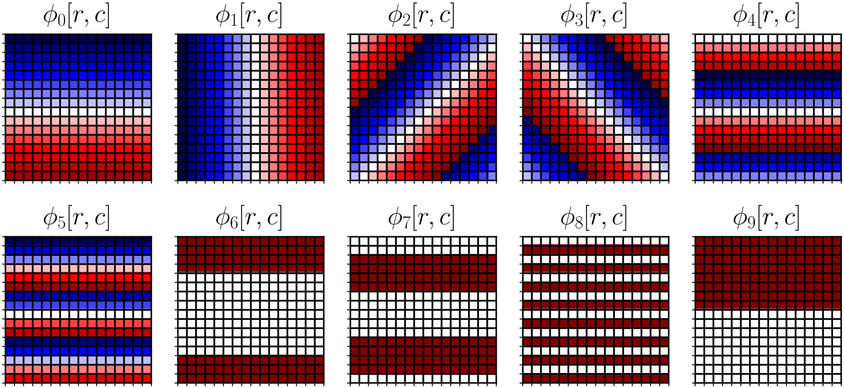
Plane Waves



The origin $(0,0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_4.

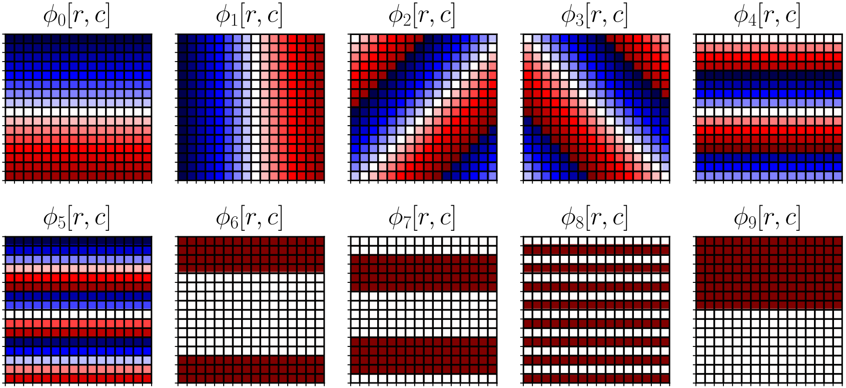
Plane Waves



The origin $(0,0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_5.

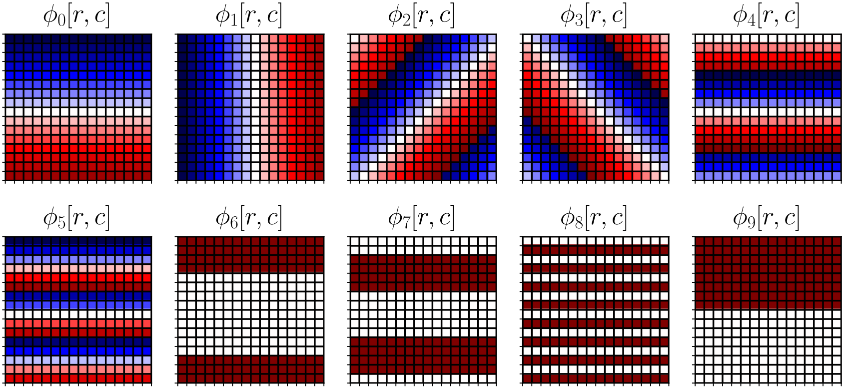
Plane Waves



The origin $(0, 0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_6.

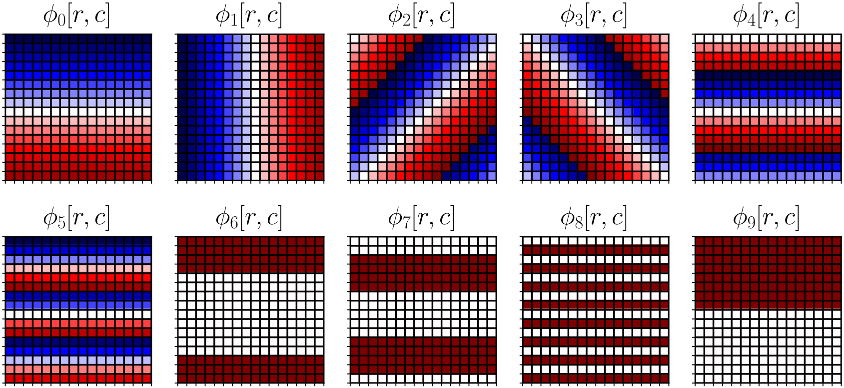
Plane Waves



The origin $(0, 0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_7.

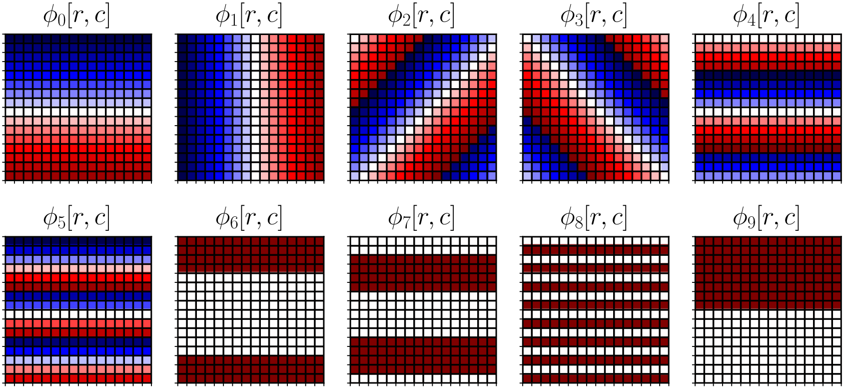
Plane Waves



The origin $(0,0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_8.

Plane Waves



The origin $(0, 0)$ lies at the center of each $R \times C$ panel. Blue denotes $-\pi$, white denotes zero, and red denotes π.

Determine a waveform with angle ϕ_9.