6.300: Signal Processing

Quiz Review

Agenda for Recitation

- Review: Fourier transforms and LTI systems

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- Review: Fourier transforms and LTI systems
- Solving problems: Quiz #2 from spring 2025

Calculus and Signal Processing

You wouldn't want to walk into a calculus quiz without knowing

$$\frac{d\sin(\theta)}{d\theta} = \cos(\theta)$$

by heart, right? Going back to the derivation wastes precious time!

$$\lim_{\phi\to 0} \frac{\sin(\theta+\phi)-\sin(\theta)}{\phi}$$

$$\lim_{\phi\to 0} \frac{\sin(\theta)\cos(\phi)+\sin(\phi)\cos(\theta)-\sin(\theta)}{\phi}$$

$$\lim_{\phi\to 0} \underbrace{\left[\frac{\sin(\phi)}{\phi}\right]}_{1 \text{ as } \phi\to 0}\cos(\theta) + \lim_{\phi\to 0} \underbrace{\left[\frac{\cos(\phi)-1}{\phi}\right]}_{0 \text{ as } \phi\to 0}\sin(\theta)$$

Calculus and Signal Processing

To do well on a **calculus** quiz,
you need to know at least a few things by heart.

common functions and their **derivatives**

$$\frac{d(t^n)}{dt} = nt^{n-1} \qquad \frac{d\big(e^{\lambda t}\big)}{dt} = \lambda e^{\lambda t} \qquad \frac{d\sin(t)}{dt} = \cos(t)$$

differentiation rules

$$\frac{d\big[c_1 f(t) + c_2 g(t)\big]}{dt} = c_1 \frac{df}{dt} + c_2 \frac{dg}{dt}$$

$$\frac{dg\big(f(t)\big)}{dt} = \frac{dg}{df} \cdot \frac{df}{dt}$$

Calculus and Signal Processing

To do well on a **signal processing** quiz,
you need to know at least a few things by heart.

common signals and their **Fourier transforms**

$$\delta(t-t_0) \iff e^{-j\omega t_0} \qquad e^{j\omega_0 t} \iff 2\pi\delta(\omega-\omega_0)$$

Fourier properties

$$c_1x_1(t)+c_2x_2(t) \iff c_1X_1(\omega)+c_2X_2(\omega)$$
$$x(t-t_0) \iff e^{-j\omega t_0}X(\omega)$$
$$e^{j\omega_0 t}x(t) \iff X(\omega-\omega_0)$$

If you're getting caught up on "level one" problems, it's difficult to do "level two" problems.

Fourier Representations

	type of time-domain signal			domain	period
CTFS	CT	period T	infinite length	integer k	none
DTFS	DT	period N	infinite length	integer k	N
CTFT	CT	aperiodic	infinite length	real ω	none
DTFT	DT	aperiodic	infinite length	real Ω	2π
DFT	DT	aperiodic	length N	integer k	N

Periodic signals have both **Fourier series** and **Fourier transform** representations.

- Fourier series coefficients: $X[k]$
- Fourier transform: $X(\omega) = \sum_k 2\pi X[k]\delta(\omega - k\omega_0)$

Duality of the continuous-time Fourier transform:

If $x(t) \iff X(\omega)$, then $X(t) \iff 2\pi x(-\omega)$, too.

Fourier Transform Pairs

	time	frequency
CTFT	$\delta(t-t_0)$	$e^{-j\omega t_0}$
CTFT	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
CTFT	$e^{-\sigma t}u(t)$ where $\sigma>0$	$1/(\sigma+j\omega)$
DTFT	$\delta[n-n_0]$	$e^{-j\Omega n_0}$
DTFT	$e^{j\Omega_0 n}$	$2\pi\delta((\Omega-\Omega_0) \bmod 2\pi)$
DTFT	$r^n u[n]$ where $\|r\|<1$	$1/(1-re^{-j\Omega})$
DFT	$\delta[n-n_0]$	$\frac{1}{N}e^{-jk\frac{2\pi}{N}n_0}$
DFT	$e^{jk_0\frac{2\pi}{N}n}$	$\delta[(k-k_0) \bmod N]$

All DTFTs are periodic in 2π. All DFTs are periodic in N.

Fourier Transform Properties

There are a few properties worth knowing by heart, too.

- A few CTFT properties are given below.
- Analogous properties apply for other representations.

	time	**frequency**
time shift	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
frequency shift	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
scaling time	$x(at)$	$\lvert a\rvert^{-1}X(a^{-1}\omega)$
scaling time	$x(a^{-1}t)$	$\lvert a\rvert X(a\omega)$
time derivative	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
frequency derivative	$t\times x(t)$	$j\frac{d}{d\omega}X(\omega)$

LTI Systems

Three equivalent representations of LTI systems:

- difference (DT) or differential (CT) equations
- unit-sample (DT) or impulse (CT) response
- frequency response

Example: System with feedback

$$y[n] = \tfrac{1}{2}y[n-1] + x[n]$$

$$h[n] = (\tfrac{1}{2})^n u[n] \iff H(\Omega) = \frac{1}{1 - \tfrac{1}{2}e^{-j\Omega}}$$

Multiple equivalent ways to perform convolution:

- directly compute the sum or integral
- superposition
- flip and shift
- transform, multiply transforms, inverse transform

Discrete Fourier Transform (DFT)

The **discrete Fourier transform (DFT)** is a discrete-time, discrete-frequency Fourier transform.

- for aperiodic discrete-time (n) signals $x[n]$
- yields discrete-frequency (k) representation $X[k]$
- finite length (N) in both time and frequency

Two equivalent perspectives of the DFT:

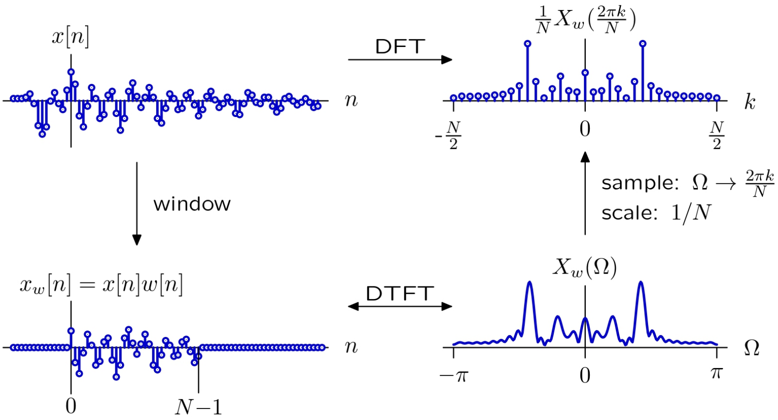
- DTFS of periodically-extended $x_w[n] = x[n]w[n]$
- Sampled ($\Omega \to 2\pi k/N$), scaled ($1/N$) DTFT of $x_w[n]$

Circular convolution: $\frac{1}{N}(x \circledast h)[n] \iff X[k]H[k]$

- Start by computing the usual convolution $(x * h)[n]$.
- Wrap into base period: $(x \circledast h)[n] = (x * h)[n \bmod N]$.
- Finally, scale by $1/N$.

Relation Between DFT and DTFT

Graphical depiction of relation between DFT and DTFT.



Graphic: Professor Denny Freeman (freeman@mit.edu)