6.300: Signal Processing

Discrete-Time Fourier Transform

Analysis: $X(\Omega)=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$

Synthesis: $x[n]=\frac{1}{2\pi}\int_{2\pi}X(\Omega)e^{j\Omega n}d\Omega$

Time Delay: $x[n-n_0] \Longleftrightarrow e^{-j\Omega n_0}X(\Omega)$

Frequency Derivative: $nx[n] \Longleftrightarrow j\frac{d}{d\Omega}X(\Omega)$

Periodic Signals: $X(\Omega)=\sum_k 2\pi X[k]\delta(\Omega-k\Omega_0)$

Agenda for Recitation

- Discrete-time Fourier transform (DTFT)

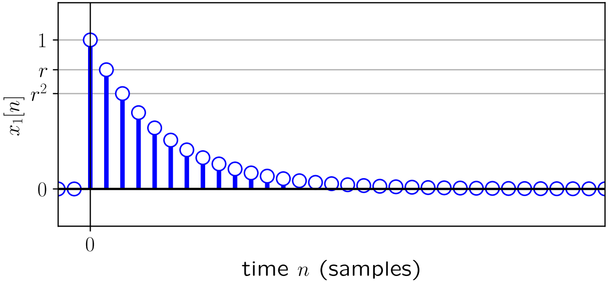
What questions do you have from lecture?

Fourier Transforms

Determine $X_1(\Omega)$, the Fourier transform of $x_1[n]$.

$$x_1[n] = \begin{cases} r^n & n \geq 0 \\ 0 & n < 0 \end{cases} \text{ where } |r| < 1$$

Sketch $|X_1(\Omega)|$ and $\angle X_1(\Omega)$.

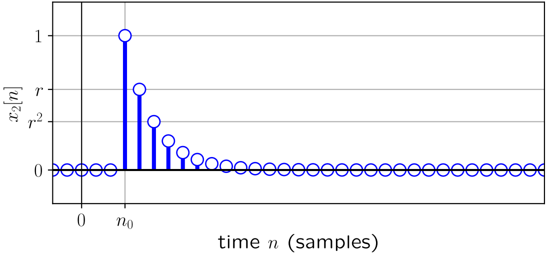


Fourier Transforms

Determine $X_2(\Omega)$, the Fourier transform of $x_2[n]$.

$$x_2[n] = x_1[n - n_0] = \begin{cases} r^{n-n_0} & n \geq n_0 \\ 0 & n < n_0 \end{cases}$$

Sketch $|X_2(\Omega)|$ and $\angle X_2(\Omega)$. How are $|X_2(\Omega)|$ and $|X_1(\Omega)|$ related? How are $\angle X_2(\Omega)$ and $\angle X_1(\Omega)$ related?

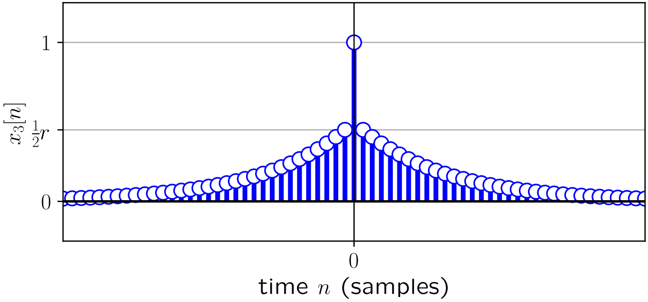


Fourier Transforms

Determine $X_3(\Omega)$, the Fourier transform of $x_3[n]$.

$$x_3[n] = \text{Symmetric}\{x_1[n]\}$$

If a time-domain signal is **real** and **symmetric**, what can you say about the Fourier transform?

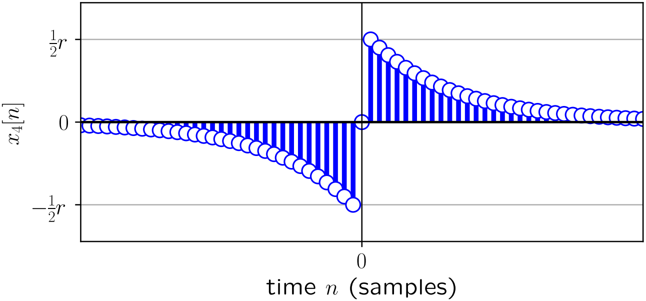


Fourier Transforms

Determine $X_4(\Omega)$, the Fourier transform of $x_4[n]$.

$$x_4[n] = \text{Anti-symmetric}\{x_1[n]\}$$

If a time-domain signal is **real** and **anti-symmetric**, what can you say about the Fourier transform?



Fourier Transforms

Determine $X_5(\Omega)$, the Fourier transform of $x_5[n]$.

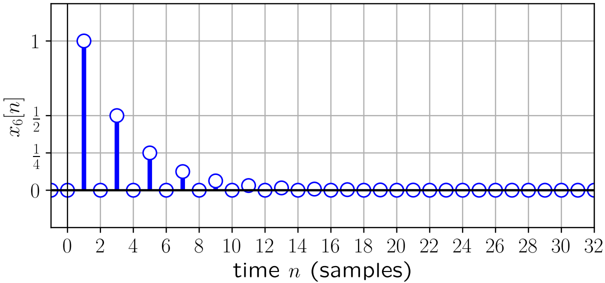
$$x_5[n] = nx_1[n]$$

Fourier Transforms

Determine $X_6(\Omega)$, the Fourier transform of $x_6[n]$.

$$x_6[n] = \begin{cases} \left(\frac{1}{2}\right)^{\frac{n}{2}} & n \in \{0, 2, 4, 6, 8, 10, 12, \dots\} \\ 0 & \text{otherwise} \end{cases}$$

What does **"stretching in time"** do in frequency?



Fourier Transforms

$x_7[n]$ has Fourier transform $X_7(\Omega)$. Determine $x_7[n]$.

$$X_7(\Omega) = e^{-j3\Omega}$$

Lessons Learned

The **discrete-time Fourier transform (DTFT)** is a Fourier representation for aperiodic and periodic discrete-time signals. It has many useful properties.

Analysis: $X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$

Synthesis: $x[n] = \frac{1}{2\pi}\int_{2\pi} X(\Omega)e^{j\Omega n}d\Omega$

Time Delay: $x[n-n_0] \iff X(\Omega)e^{-j\Omega n_0}$

Frequency Derivative: $nx[n] \iff j\frac{d}{d\Omega}X(\Omega)$

Periodic Signals: $X(\Omega) = \sum_k 2\pi X[k]\delta(\Omega - k\Omega_0)$