

6.300: Signal Processing

Continuous-Time Fourier Transform

Analysis: $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

Synthesis: $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

Time Delay: $x(t - t_0) \iff X(\omega) e^{-j\omega t_0}$

Time Derivative: $\frac{d}{dt} x(t) \iff j\omega X(\omega)$

Periodic Signals: $X(\omega) = \sum_k 2\pi X[k] \delta(\omega - k\omega_0)$

Agenda for Recitation

- Continuous-time Fourier transform (CTFT)

What questions do you have from lecture?

Fourier Transform Conventions

There are many competing conventions for how to define the Fourier transform. **You can refer to any source you want, but you must use our conventions in 6.300!**

$$X(\omega) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \qquad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

Don't use these conventions — at least for now!

$X(j\omega) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ Why call it a function of $j\omega$?

$X(f) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$ Function of f , not $\omega = 2\pi f$.

$\hat{x}(\omega) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$ Too many $\sqrt{2\pi}$ factors.

While we're all still learning the basics, please stick to our conventions. We want to minimize confusion.

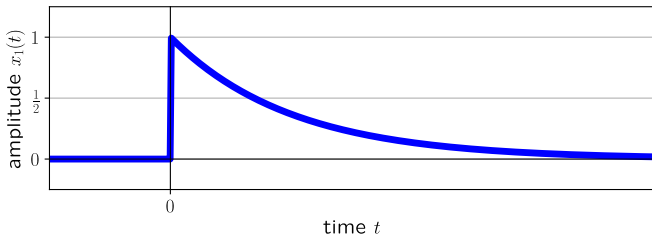
Fourier Transforms

Determine $X_1(\omega)$, the Fourier transform of $x_1(t)$.

$$x_1(t) = \begin{cases} e^{-t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Next, sketch $\text{Re}\{X_1(\omega)\}$ and $\text{Im}\{X_1(\omega)\}$.

Finally, sketch $|X_1(\omega)|$ and $\angle X_1(\omega)$.

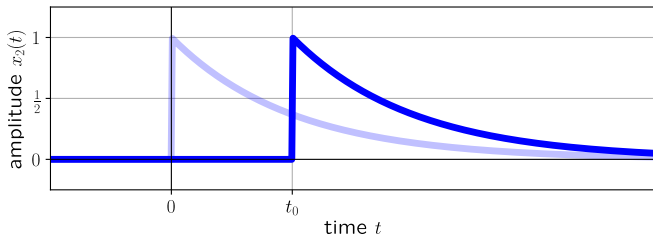


Fourier Transforms

Determine $X_2(\omega)$, the Fourier transform of $x_2(t)$.

$$x_2(t) = x_1(t - t_0) = \begin{cases} e^{-(t-t_0)} & t \geq t_0 \\ 0 & t < t_0 \end{cases}$$

Sketch $|X_2(\omega)|$ and $\angle X_2(\omega)$. How are $|X_2(\omega)|$ and $|X_1(\omega)|$ related? How are $\angle X_2(\omega)$ and $\angle X_1(\omega)$ related?

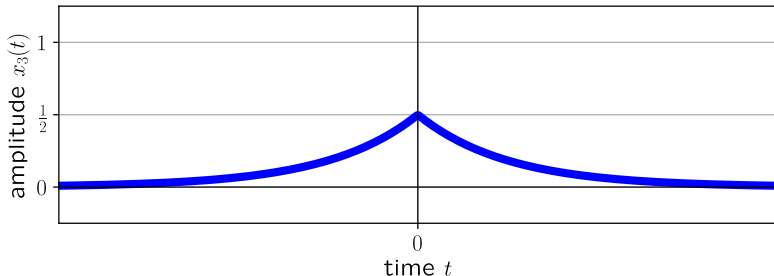


Fourier Transforms

Determine $X_3(\omega)$, the Fourier transform of $x_3(t)$.

$$x_3(t) = \text{Symmetric}\{x_1(t)\}$$

If a time-domain signal is **real** and **symmetric**,
what can you say about the Fourier transform?

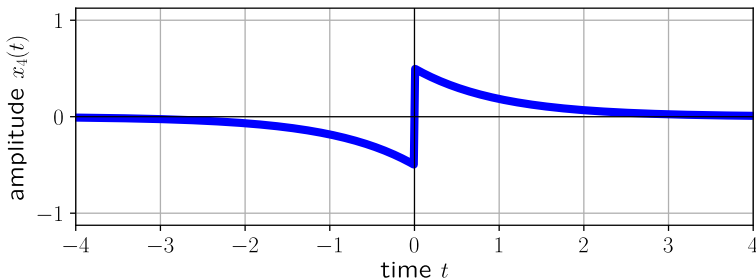


Fourier Transforms

Determine $X_4(\omega)$, the Fourier transform of $x_4(t)$.

$$x_4(t) = \text{Anti-symmetric}\{x_1(t)\}$$

If a time-domain signal is **real** and **anti-symmetric**, what can you say about the Fourier transform?



Fourier Transforms

Determine $X_5(\omega)$, the Fourier transform of $x_5(t)$.

$$x_5(t) = \frac{d}{dt}x_4(t)$$

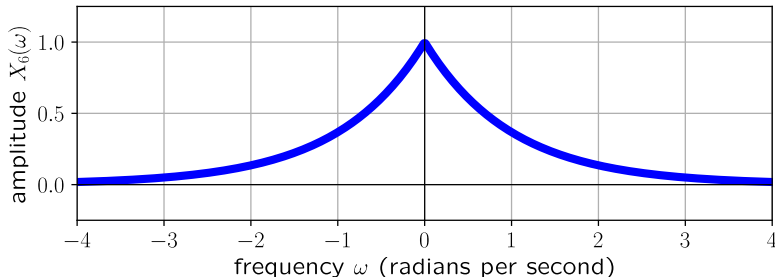
Hint: Start from the **synthesis equation**.

What does **differentiating with respect to time** do to the Fourier transform of a signal?

Fourier Transforms

$x_6(t)$ has Fourier transform $X_6(\omega)$. Determine $x_6(t)$.

$$X_6(\omega) = e^{-|\omega|} = \begin{cases} e^{-\omega} & \omega \geq 0 \\ e^{\omega} & \omega < 0 \end{cases}$$



Lessons Learned

The **continuous-time Fourier transform (CTFT)** is a Fourier representation for aperiodic and periodic continuous-time signals. It has many useful properties.

Analysis: $X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

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