6.300: Signal Processing

Fourier Series II: Symmetry

The **synthesis equation** tells us how to represent a periodic signal as a Fourier series.

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(k\frac{2\pi}{T}t\right) + \sum_{k=1}^{\infty} d_k \sin\left(k\frac{2\pi}{T}t\right)$$

The **analysis equations** tell us how to calculate these Fourier series coefficients.

$c_0 = \frac{1}{T}\int_T f(t)\,dt$ average value over a period

$c_k = \frac{2}{T}\int_T f(t)\cos\left(k\frac{2\pi}{T}t\right)dt$ for $k \geq 1$

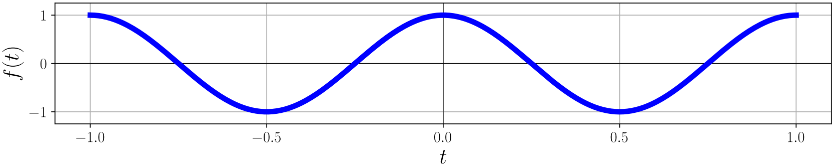
$d_k = \frac{2}{T}\int_T f(t)\sin\left(k\frac{2\pi}{T}t\right)dt$ for $k \geq 1$

Agenda for Recitation

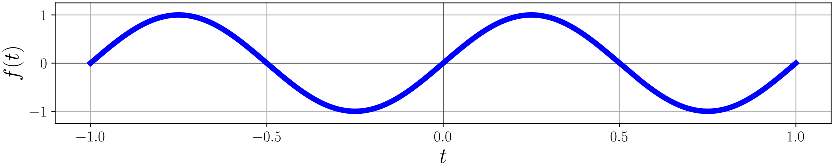
- Definitions of **symmetry**, **anti-symmetry**, **asymmetry**
- Using **symmetry** to simplify Fourier series calculations

Symmetry, Anti-Symmetry, Asymmetry

A **symmetric** signal satisfies $f(t) = f(-t)$.



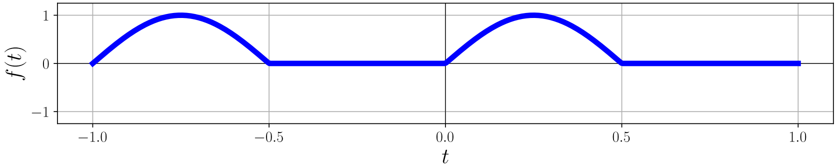
An **anti-symmetric** signal satisfies $f(t) = -f(-t)$.



What we've called **symmetry/anti-symmetry** is also called **even/odd symmetry**. Might as well know both.

Symmetry, Anti-Symmetry, Asymmetry

An **asymmetric** signal is neither symmetric nor anti-symmetric — it lacks any semblance of symmetry.



We can express an **asymmetric** signal as the **sum** of a **symmetric part** and an **anti-symmetric part**.

$$f(t) = f_{\text{symmetric}}(t) + f_{\text{anti-symmetric}}(t)$$

How do we get $f_{\text{symmetric}}(t)$ from $f(t)$?
How do we get $f_{\text{anti-symmetric}}(t)$ from $f(t)$?

Symmetry, Anti-Symmetry, Asymmetry

Before we make any calculations, we can use **symmetry** (and **anti-symmetry**) to make some conclusions about the Fourier series coefficients.

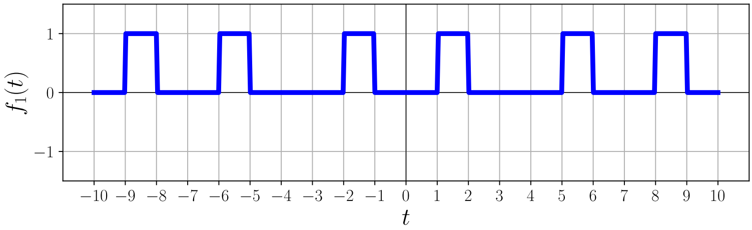
What can we say about the Fourier series coefficients (c_k and d_k) for a **real, symmetric** signal?

What can we say about the Fourier series coefficients (c_k and d_k) for a **real, anti-symmetric** signal?

Instead of keeping the discussion abstract, let's work through examples and draw some conclusions from there.

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_1(t)$, shown below.



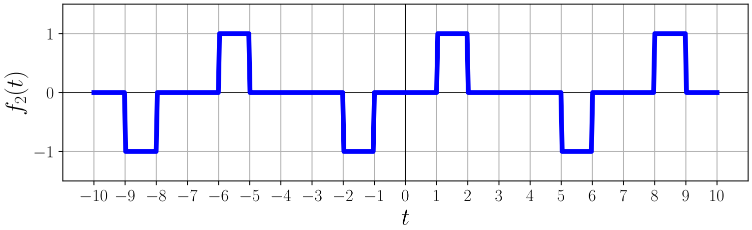
What is the fundamental period (T) of $f_1(t)$?

Does it matter which interval you integrate over?
(0 to T? $-\frac{1}{2}T$ to $\frac{1}{2}T$? $\frac{5}{16}T$ to $\frac{21}{16}T$?)

Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

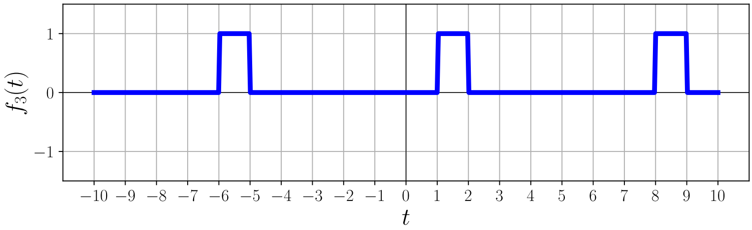
Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_2(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

Fourier Series and Symmetry

Determine the Fourier series coefficients (c_k and d_k) for the periodic function $f_3(t)$, shown below.



Do you notice anything about the c_k and/or d_k terms?

How does $f_3(t)$ relate to $f_1(t)$ and $f_2(t)$?

How do the Fourier series coefficients for $f_3(t)$ relate to the coefficients for $f_1(t)$ and $f_2(t)$?

Fourier Series and Symmetry

We have looked at how to use **symmetry** to simplify calculations with Fourier series.

Real, symmetric signals: Only c_k are non-zero.
Real, anti-symmetric signals: Only d_k are non-zero.

We will soon learn many more **Fourier properties**.

e.g., If we know c_k/d_k for $f(t)$, what about c_k/d_k for ...

- $f(t-1)$?
- $f(2t)$?
- $f(t)\cos(t)$?
- $f^2(t)$?

Next time: Simplifying the math even further using **complex numbers — replacing trig with algebra!**

Lessons Learned

The **synthesis equation** tells us how to represent a periodic signal as a Fourier series.

$$f(t)=f(t+T)=c_0+\sum_{k=1}^{\infty} c_k \cos\left(k\tfrac{2\pi}{T}t\right)+\sum_{k=1}^{\infty} d_k \sin\left(k\tfrac{2\pi}{T}t\right)$$

The **analysis equations** tell us how to calculate these Fourier series coefficients.

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$d_k=\frac{2}{T}\int_T f(t)\sin\left(k\frac{2\pi}{T}t\right)dt$ — for $k\geq 1$

Real, symmetric signals: Only c_k are non-zero.

Real, anti-symmetric signals: Only d_k are non-zero.