Dimensional Analysis

Quantity	Symbols	Dimensions
angle	θ	none
length	$d, D, h, H, \ell, L, r, R$	L
area	A	L^2
volume	$V, \mathcal{V}$	L^3
time	t, T	T
velocity	c, u, U, v	LT^{-1}
acceleration	$\dot{u}, \dot{U}, \dot{v}$	LT^{-2}
angular velocity	ω, Ω	T^{-1}
mass density	ρ	ML^{-3}
momentum	$\mathcal{L}$	MLT^{-1}
volume flow rate	$Q_{\text{vol}} = \iint_S (\vec{v} \cdot \hat{n})\,\mathrm{d}S$	L^3T^{-1}
mass flow rate	$Q_{\text{mass}} = \rho Q_{\text{vol}}$	MT^{-1}
pressure	p, P	$ML^{-1}T^{-2}$
stress	τ	$ML^{-1}T^{-2}$
surface tension	Σ	MT^{-2}
force	F	MLT^{-2}
moment	I, M	ML^2T^{-2}
energy	E, W	ML^2T^{-2}
power	$\dot{E}, P, \dot{W}$	ML^2T^{-3}
dynamic viscosity	$\mu = \rho\nu$	$ML^{-1}T^{-1}$
kinematic viscosity	$\nu = \mu\rho^{-1}$	L^2T^{-1}

Common Similarity Parameters

Number	Ratio	Definition
cavitation	$F_{\text{pressure}} \div F_{\text{inertial}}$	$(p_0 - p_v) \div (\frac{1}{2}\rho U^2)$
Euler	$F_{\text{pressure}} \div F_{\text{inertial}}$	$p_0 \div (\frac{1}{2}\rho U^2)$
Froude	$F_{\text{inertial}} \div F_{\text{gravity}}$	$U \div \sqrt{gL}$
Mach	$U_{\text{flow}} \div U_{\text{sound}}$	$U \div c$
Reynolds	$F_{\text{inertial}} \div F_{\text{viscous}}$	$(\rho U L) \div \mu$
Strouhal	$F_{\text{Eulerian}} \div F_{\text{convective}}$	$L \div (UT)$
Weber	$F_{\text{inertial}} \div F_{\text{surface tension}}$	$(\rho U^2 L) \div \Sigma$
drag (or lift)	$F_{\text{drag}} \div F_{\text{dynamic}}$	$F_{\text{drag}} \div (\frac{1}{2}\rho U^2 A)$

Navier-Stokes (Conservation of Momentum)

$$\frac{D\vec{v}}{Dt} = -\frac{\nabla p}{\rho} + \nu\nabla^2\vec{v} + \frac{\vec{F}}{\rho}$$

$$\frac{\partial u_i}{\partial t} + \left(u\frac{\partial u_i}{\partial x} + v\frac{\partial u_i}{\partial y}\right) = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu\left(\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2}\right) + \frac{F_i}{\rho}$$

Continuity: $\nabla \cdot \vec{v} = 0$

KBC: $\vec{v}_{\text{fluid}} = \vec{v}_{\text{boundary}}$ on the boundary (no slip, no flux)

DBC: $\Delta p = p_{\text{in}} - p_{\text{out}}$ and $\tau_{ij}^{\text{interface}} = \tau_{ij}^{\text{out}} - \tau_{ij}^{\text{in}}$

Bernoulli Equation (from Euler Equations)

Incompressible, inviscid, steady flow: $\nabla \cdot \vec{v} = 0$, $\nu = 0$, $\partial_t \vec{v} = \vec{0}$

$$\frac{|\vec{v}_1|^2}{2} + \frac{p_1}{\rho} + gh_1 = \frac{|\vec{v}_2|^2}{2} + \frac{p_2}{\rho} + gh_2 \text{ (along streamline)}$$

Quantity	Definition
material derivative	$\frac{D\vec{v}}{Dt} \triangleq \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla\vec{v}$
streamlines	$u^{-1}\mathrm{d}x = v^{-1}\mathrm{d}y = w^{-1}\mathrm{d}z$
stress tensor	$\tau_{ij} = \tau_{ji}$ (direction i, orientation j)
viscous stress	$\hat{\tau}_{11} = 2\mu(\partial_x u), \hat{\tau}_{12} = \mu(\partial_y u + \partial_x v)$
force on surface	$F_i = A\tau_i$ where $\tau_i \triangleq \tau_{ij} n_j$
total pressure	$p = p_{\text{d}} + p_{\text{s}}$ (gravity: $p_{\text{s}} = -\rho g z$)
$\Delta p_{\text{interface}}$ (general)	$\Delta p = p_{\text{in}} - p_{\text{out}} = \Sigma\,(R_1^{-1} + R_2^{-1})$
$\Delta p_{\text{interface}}$ (bubbles)	$\Delta p = 2\frac{\Sigma}{R}$ (gas) or $\Delta p \approx 4\frac{\Sigma}{R}$ (soap)
$\vec{v}_{\text{Couette}}$ (velocity)	$u(y) = \frac{1}{2\mu}(-\frac{dp}{dx})(h-y)y + U_{\text{top}}\frac{y}{h}$
τ_{Couette} (stress)	$\tau_{\text{bottom wall}} = \frac{1}{2}h(-\frac{dp}{dx}) + U_{\text{top}}\frac{\mu}{h}$
$\vec{v}_{\text{Poiseuille}}$ (velocity)	$v_x(r) = \frac{1}{4\mu}(-\frac{dp}{dx})(a^2 - r^2)$
$Q_{\text{Poiseuille}}$ (volume flow)	$Q = \frac{\pi}{8\mu}a^4(-\frac{dp}{dx})$
$\tau_{\text{Poiseuille}}$ (stress)	$\tau_{\text{wall}} = -\tau_{xy} = \frac{a}{2}(-\frac{dp}{dx})$
vorticity	$\vec{\omega} \triangleq \nabla \times \vec{v}$
vorticity in 2D	$\vec{\omega} = (\omega_x, \omega_y, \omega_z) = (0, 0, \partial_x v - \partial_y u)$
circulation ($\vec{\omega}$ flux)	$\Gamma \triangleq \oint_C \vec{v} \cdot \mathrm{d}\vec{x} = \iint_S \vec{\omega} \cdot \vec{n}\,\mathrm{d}S$

Fluid Flow Assumptions

- steady flow: $\partial\vec{v}/\partial t = \vec{0}$ (Eulerian)
- incompressible flow: $D\rho/Dt = 0$ or $\nabla \cdot \vec{v} = 0$ (Lagrangian)
- inviscid flow: $\nu = 0$
- ideal flow: incompressible, inviscid ("no slip" doesn't apply)
- Kelvin's theorem says that angular momentum is conserved for ideal flow under conservative forces, or IFCF: $d\Gamma/dt = 0$ for any material contour C.

Control Volume Analysis

1. Specify control volume. (e.g., Along a streamline, $\vec{v} \cdot \vec{n} = 0$.)
2. For each surface, define $\mathrm{d}S$, $\vec{n}$, $\vec{v}$, $\vec{F}_{\text{surface}}$, $(\vec{v} \cdot \vec{n})$, and $\vec{v}\,(\vec{v} \cdot \vec{n})$. Apply boundary conditions (e.g., "no flux") as needed.
3. Determine volume integral quantities: $\vec{F}_{\text{body}}$, $\frac{\partial \rho}{\partial t}$, and $\frac{\partial \vec{v}}{\partial t}$.
4. Apply conservation of mass (continuity) to each surface.

$$\iiint_V \frac{\partial \rho}{\partial t}\,\mathrm{d}V + \oiint_S \rho\,(\vec{v} \cdot \vec{n})\,\mathrm{d}S = 0$$

5. Apply conservation of momentum to each surface.

$$\iiint \frac{\partial(\rho v_i)}{\partial t}\,\mathrm{d}V = -\oiint_S \rho\, v_i\,(v_j n_j)\,\mathrm{d}S + \iiint_V F_{\text{body}_i}\,\mathrm{d}V - \oiint_S p\, n_i\,\mathrm{d}S + \oiint_S \hat{\tau}_{ij} n_j\,\mathrm{d}S$$

where $\tau_{ij} = -p\,\delta_{ij} + \hat{\tau}_{ij}$ denotes the stress tensor

Kinematic Transport Theorems

f is a per-volume fluid property. G is a per-mass fluid property.

$$\frac{d}{dt}\iiint_{V(t)} f(\vec{x}, t)\,\mathrm{d}V = \iiint_{V(t)} \frac{\partial f(\vec{x}, t)}{\partial t}\mathrm{d}V + \iint_{S(t)} f(\vec{x}, t)\,U_{\text{normal}}(\vec{x}, t)\,\mathrm{d}S$$

$$\frac{d}{dt}\iiint_{V_{\text{mat}}(t)} f(\vec{x}, t)\,\mathrm{d}V = \iiint_{V_{\text{mat}}(t)} \left[\frac{\partial f(\vec{x}, t)}{\partial t} + \nabla \cdot (f\vec{v})\right]\mathrm{d}V \qquad (1^{\underline{\text{st}}})$$

$$\frac{d}{dt}\iiint_{V_{\text{mat}}} \rho G\,\mathrm{d}V = \iiint_{V_{\text{mat}}} \rho\frac{DG}{Dt}\mathrm{d}V \qquad (2^{\underline{\text{nd}}})$$

$1^{\underline{\text{st}}}$ KTT: Assume that the control volume is a material volume.
$2^{\underline{\text{nd}}}$ KTT: Use the $1^{\underline{\text{st}}}$ KTT and assume conservation of mass.