

ES.S20: Group Theory

Worksheet #3

Problem #1

- (i) Write down one example of a group of order 7.
- (ii) Show that all groups of order 7 are identical with the one you wrote down in (i).

Problem #2

Can you generalize what you just did in **Problem #1**? Hint: Does this apply to more integers than just 7?

Problem #3

Write down all you currently know about groups of order $m \in \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Problem #4

Write out the multiplication table for $\mathbb{Z}_5$, where $\mathbb{Z}_5$ denotes the integers modulo 5 under addition. Next, write out the multiplication table for $\mathbb{Z}_5^*$, where $\mathbb{Z}_5^*$ denotes the non-zero integers modulo 5 under multiplication.

- (i) Find an element of order 5 in $\mathbb{Z}_5$.
- (ii) Find an element of order 4 in $\mathbb{Z}_5^*$.

$\mathbb{Z}_5$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{0}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{0}$
$\overline{2}$	$\overline{2}$	$\overline{3}$	$\overline{4}$	$\overline{0}$	$\overline{1}$
$\overline{3}$	$\overline{3}$	$\overline{4}$	$\overline{0}$	$\overline{1}$	$\overline{2}$
$\overline{4}$	$\overline{4}$	$\overline{0}$	$\overline{1}$	$\overline{2}$	$\overline{3}$

$\mathbb{Z}_5^*$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{1}$	$\overline{1}$	$\overline{2}$	$\overline{3}$	$\overline{4}$
$\overline{2}$	$\overline{2}$	$\overline{4}$	$\overline{1}$	$\overline{3}$
$\overline{3}$	$\overline{3}$	$\overline{1}$	$\overline{4}$	$\overline{2}$
$\overline{4}$	$\overline{4}$	$\overline{3}$	$\overline{2}$	$\overline{1}$